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ASYMPTOTIC INVESTIGATION OF NONLINEAR EFFECTS IN THE PROBLEM OF UNSTEADY SUPERSONIC FLOW PAST A PROFILE

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It is shown that the problem of unsteady flow of a supersonic gas stream past a slender profile has, under certain conditions, an irregular expansion in the neighborhood of the bow shock wave. A nonlinear equation which defines the flow in the irregularity region is obtained, and the linear and nonlinear expansions are matched. Equations of general form are derived for the flow in the neighborhood of weak shock waves which in the linear theory coincide with characteristics.

1. We consider in linear formulation the class of problems of supersonic flow in which the weakly perturbed flow region is separated from that of uniform flow by the surface $\xi = x - \beta y = 0$ ($\beta = (M^2 - 1)^{1/2}$, $M = Ua^{-1}$; M and a are, respectively, the Mach number and the speed of sound in the uniform stream. Such flows occur, for instance, in supersonic flows past an unsteady point source of small perturbations, in the supersonic flow around a pointed slender profile, or some insignificant convexity on a plane or cylindrical surface (see, e.g., [1 —5] dealing with unsteady supersonic flows and provide surveys of these problems).

We specify the linear expansion of the velocity potential in the form

$$\Phi(x, y, t) = Ux + \delta\varphi_1(\xi, y, t) + \delta^2\varphi_2(\xi, y, t) + \ldots \qquad (1.1)$$
$$\xi = x - \beta y$$

where Φ, x, y, and t are respectively, the dimensionless velocity potential, the coordinates, and time, normalized with respect to $a_*^2 t_0$, $a_* t_0$, and t_0, where a_* is the speed of sound in the sonic stream.

Conditions of the problem are assumed to be such that in the neighborhood of the perturbed region boundary $\xi = 0$ the potential φ_1 is of the form

$$\varphi_1 = E(y, t)\xi^m \ln^n \xi + T(y, t) g(\xi) + \ldots, \quad g(\xi) \ll \xi^m \ln^n \xi \qquad (1.2)$$

The values of m and n must be such that $\varphi_1(\xi = 0) = 0$. For $E(y, t)$ from the linear equation for φ_1 we then have

$$2UE_t + a^2\beta\left(2E_y + \frac{\omega}{y} E\right) = 0, \quad E = E_0(\eta) y^{-\omega/2}, \quad \eta = y - \frac{a\beta}{M} t \qquad (1.3)$$

where $E_0(\eta)$ is an arbitrary function, and $\omega = 0$ and $\omega = 1$ for plane and axisymmetric flows, respectively.

Expansion (1.2) corresponds, for example, to flow over a surface for which in the linear theory for small x (at the profile nose $x = y = 0$) is defined by

$$y = \delta y_0(x, t) = \delta h(t) x^m \ln^n x + h_1(t) g(x) + \ldots \qquad (1.4)$$

The external flow past a cylindrical surface $y = y^* + \delta y_0(x, t)$, $y^* = \text{const} \neq 0$ may be considered as an example of axisymmetric flows. In that case the variable ξ is to be specified in the form $\xi = x - \beta(y - y^*)$. It follows from (1.4) that the profile is pointed when $m = 1$, $n \leqslant 0$, or $m > 1$. These values are used below. It follows from (1.2) that when $m = 1$ and $n = 0$ (the angle α between the tangent to the profile and the x-axis at $x = 0$ is nonzero), then in linear formulation $\xi = 0$ the shock wave intensity is $\Delta P \sim \delta$: when $m = 1$ and $n < 0$, or $m > 1$ ($\alpha = 0$), the potential and velocities at $\xi = 0$ are continuous and $\xi = 0$ is a characteristic that represent the surface of weak discontinuity.

Note that for certain m, n, and $g(\xi)$ we have $\varphi_{1\xi\xi}(\xi = 0) = \infty$. If on the characteristic $\xi = 0$ we have $\Phi = Ux$, then the exact equation of the potential $\Phi(\xi, y, t)$ yields for the determination of $z(y, t) = \Phi_{\xi\xi}(\xi = 0)$ the nonlinear equation

$$2Uz_t + a^2\beta\left(2z_y + \frac{\omega}{y}z\right) + (\varkappa + 1)UM^2z^2 = 0 \tag{1.5}$$

It follows from (1.5) that $\Phi(\xi = 0) = Ux + z\xi^2$ and that the acceleration $\Phi_{\xi\xi}(\xi = 0)$ is either a quantity of order unity $(z = z(y, t) \neq 0)$ or is equal zero $(z = 0)$. However the linear theory does not provide the possibility of satisfying the condition of finiteness of acceleration (in the linear theory $\Phi_{\xi\xi} = \delta\varphi_{1\xi\xi} \sim \delta \ll 1$) and this is the cause of $\varphi_{1\xi\xi}(\xi = 0)$ becoming ∞ in such cases. The linear theory assumes acceleration $\Phi_{\xi\xi}$ to be small and defines $\varphi_{1\xi\xi}(\xi = 0)$ by the linear equation (1.5) without the last term. Similar investigations were carried out in [6] for conical steady flows.

The two-term linear expansion (external in [7 – 9]) in the neighborhood of $\xi = 0$ for $m > 1$ or $m = 1$ and $n < 0$ ($\alpha = 0$), is of the form

$$\begin{gathered}\Phi = Ux + \delta\left[E_0(\eta)\, y^{-\omega/2}\xi^m \ln^n \xi + \ldots\right] + \\ \delta^2\left\{\left[E_1(\eta) - \frac{pm^2}{2}E_0{}^2(\eta)\, y^{(2-\omega)/2}\right] y^{-\omega/2}\xi^{2m-2}\ln^{2n}\xi + \ldots\right\} \\ p = (\varkappa + 1)M^3(2 - \omega)^{-1}(a\beta)^{-1}\end{gathered} \tag{1.6}$$

where $\xi \to 0$, y and $t \sim 1$, and values of M are finite and not close to unity.

For $m = 1$ and $n = 0$ ($\alpha \neq 0$) we have

$$\Phi = Ux + \delta\left[E_0(\eta)\, y^{-\omega/2}\,\xi + \ldots\right] + \delta^2\left\{\left[\nu\xi + \gamma g'(\xi)\right] + \ldots\right\} \tag{1.7}$$

where $E_1(\eta)$ is an arbitrary function; $\nu(y, t)$ and $\gamma(y, t)$ are some functions of variables y and t. In (1.7) the term containing ν is to be discarded when $\xi^2 \ll g \ll \xi$; when $g \ll \xi^2$ the term containing γ is to be eliminated, and when $g \sim \xi^2$ both terms are to be retrained. Expansions (1.6) and (1.7) are irregular when $\delta\xi^{m-2}\ln^n\xi \sim 1$ (if $n = 0$ then for $\xi \sim \delta^{1/(2-m)}$) and for $\xi^2 \ll g \ll \xi$, $\xi \sim \delta g'(\xi)$, respectively. Specifically, irregularity occurs for those m, n, and $g(\xi)$ for which $\varphi_{1\xi\xi}(\xi = 0) = \infty$.

We have the irregularity of linear expansion in the neighborhood of shock waves (characteristics) where values of parameters are small but their gradients are finite [6 – 14].

2. We specify the expansion (internal in [7 – 9] that defines the flow in the

neighborhood of a shock wave (characteristic) by

$$\Phi = Ux + \varepsilon^2\psi(\xi^\circ, y, t) + \ldots, \quad \xi = \varepsilon\xi^\circ, \quad \varepsilon \ll 1 \tag{2.1}$$

Assuming that y and $t \sim 1$, and $1 < M < \infty$, we obtain for ψ the equation

$$2U\psi_{\xi^\circ t} + a^2\beta\left(2\psi_{\xi^\circ y} + \frac{\omega}{y}\psi_{\xi^\circ}\right) + (\varkappa + 1)UM^2\psi_{\xi^\circ}\psi_{\xi^\circ\xi^\circ} = 0 \tag{2.2}$$

which for the determination of $\Phi_{\xi\xi}$ ($\xi = 0$) yields exactly Eq. (1.5) with $\Phi_{\xi\xi} \sim 1$. Equation (2.2) was considered in [7] for stable plane flows.

At the shock front the conditions are of the form

$$\frac{\varkappa+1}{2}UM^2(\psi_{\xi^\circ} + \psi^*_{\xi^\circ}) = 2a^2\beta\frac{\partial\xi^\circ}{\partial y} + 2U\frac{\partial\xi^\circ}{\partial t} \tag{2.3}$$
$$\psi = \psi^* \quad \text{for} \quad \xi^\circ = \xi^\circ(y, t)$$

where ψ^* relates to the flow ahead of the shock wave, and when $\psi^* \equiv \psi$, (2.3) is the characteristic equation.

The impenetrability condition along the profile $y = \varepsilon^2 y_0(\xi, t)$ for $\omega = 0$ is of the form $U\partial y_0 / \partial\xi^\circ = -\beta\psi_{\xi^\circ}$ at $y = 0$. For $\omega = 1$ the profile is to be specified in the form $y = y^* + \varepsilon^2 y_0$, $y^* = \text{const} \neq 0$.

The general solution of Eq. (2.2)

$$\xi^\circ = pu^\circ y + f(u^\circ y^{\omega/2}, \eta), \quad u^\circ = \psi_{\xi^\circ}, \quad \Phi_\xi = U + \varepsilon u^\circ \tag{2.4}$$

Along the characteristics of Eq. (2.2) the relation

$$\left(py + \frac{\partial f}{\partial\mu}y^{\omega/2}\right)\left(2a^2\beta\frac{\partial u^\circ}{\partial y} + 2U\frac{\partial u^\circ}{\partial t} + \frac{\omega}{y}a^2\beta u^\circ\right) = 0 \tag{2.5}$$

where $u^\circ = u^\circ(y, t)$, is the value of u° on the characteristic and $\mu = u^\circ y^{\omega/2}$, is valid. Then along the characteristic $\xi^\circ = pu_0(\eta)y^{(2-\omega)/2} + f[u_0(\eta), \eta]$ velocity u° is determined by (1.3) and $u^\circ = u_0(\eta)y^{-\omega/2}$ ($u_0(\eta)$ is determined by the condition at the profile). Equation $py + (\partial f / \partial\mu)y^{\omega/2} = 0$ in conjunction with (2.4) determines the envelope of the indicated set of characteristics, as well as that of the set of curves (2.4), where u° is a parameter which is constant along each of these curves. Note that equation $\eta = 0$ determines the coordinate of the intersection point of the straight line $\xi = 0$ with the boundary of the sound momentum $(x - Ut)^2 + y^2 = a^2t^2$.

3. Passing in the linear equation for φ_1 to the internal variable ξ° and in Eq. (2.2) to the external variable ξ, we obtain for $\varepsilon \to 0$ the same limit equation, which indicates the possibility of joining expansions (1.1) and (1.2). Let us amend the linear theory in the neighborhood of $\xi = 0$ with $\alpha = 0$ (see (1.6)) and $n = 0$. In this case $1 < m < 2$, since the irregularity region $\xi \sim \delta^{1/(2-m)} \ll 1$ (it is interesting that when $\xi^\circ \to \infty$, solution of (2.2) of the form (1.2) exists only when $m < 2$ or $m = 2$ and $n \leqslant 0$). Joining the one-term expansions (1.6), (2.1), and (2.4), we obtain $\varepsilon = \delta^{1/(2-m)}$ and $f^{m-1} = u^\circ m^{-1}E_0^{-1}y^{\omega/2}$. Let us consider the particular case in which f is a second power polynomial in powers of u°

$$\psi = A(\xi^\circ + B)^{3/2} + C\xi^\circ + D, \quad Cy^{\omega/2} = C_0 - \frac{9}{8}pA_0^2y^{(2-\omega)/2} \tag{3.1}$$
$$A = A_0y^{-\omega/2}, \quad B = B_0 - pC_0y^{(2-\omega)/2} + \frac{9}{16}p^2A_0^2y^{2-\omega}$$

where $D(y, t)$, $A_0(\eta)$, $B_0(\eta)$, and $C_0(\eta)$ are arbitrary functions. For (3.1) $\Phi_{\xi\xi}(\xi = 0) = {}^3/_4 AB^{-1/2}$. Joining with (1.6) yields $\varepsilon = \delta^2$, $m = {}^3/_2$, $A_0 = E_0$, and $C_0 = E_1$. The condition of impenetrability implies that the equation of the profile for (3.1) for small x is of the form $(\omega = 0)$

$$U\beta^{-1}y = -\sqrt{\varepsilon}\, A_0(t^*)x^{3/2} - \varepsilon C_0(t^*)\, x - {}^3/_2\varepsilon\, \beta A_0{}^2 x^2 + \ldots, \quad a\beta M^{-1}t = -t^*$$

Using (2.3) it can be easily shown that solution (3.1) is suitable for defining the flow in which the perturbed region is bounded upstream by the characteristic $\xi^\circ = {}^4/_9 C_0{}^2 A_0{}^{-2} - B_0$ $(\omega = 0, 1)$ along which it is not discontinuous, or by the shock wave

$$\xi^\circ = \xi^* = -B_0 + \frac{1}{9} C_0{}^2 A_0{}^{-2} + \frac{3}{4} pC_0 y^{(2-\omega)/2} - \frac{27}{64} p^2 A_0{}^2 y^{2-\omega}, \quad \omega = 0, 1$$

If the characteristic (shock wave) passes through point $x = y = 0$, then $B_0 = {}^4/_9 C_0{}^2 A_0{}^{-2}$ $(B_0 = {}^1/_9 C_0{}^2 A_0{}^{-2})$, and its intensity is

$$\frac{\Delta P}{\varkappa M} a^{-(\varkappa+1)/(\varkappa-1)} = -\varepsilon\psi_{\xi^\circ}(\xi^\circ = \xi^*) = \varepsilon\left(\frac{27}{16} pA_0{}^2 y^{1-\omega} - \frac{3}{2} C_0 y^{-\omega/2}\right)$$

If the shock wave originates at the profile nose, then $C_0 \leqslant 0$ (otherwise ΔP $(y = 0) < 0$). If $C_0 = 0$, the shock wave intensity at $y = 0$ $(\omega = 0)$ is zero. The compression shock at the profile is obtained for $\omega = 1$ by setting in the expression for ΔP $y = y^* \neq 0$.

In linear formulation the shock wave intensity $\Delta P \sim \delta$. In the considered particular cases $\Delta P \sim \varepsilon = \delta^{1/(2-m)} \lll \delta$ $(1 < m < 2)$ for $n = 0$. Hence in such formulation the shock wave of such weak intensity degenerates into a characteristic, i.e., a surface of weak discontinuity.

A similar investigation of the plane problem of steady supersonic flow past a profile for $\alpha = n = 0$ was carried out in [14].

4. In conformity with [13] Eq. (2.2) can be extended to the case of a viscous heat conducting gas. Specifying $P = P_0$, $\rho = \rho_0$, $\Pr = \Pr_0$, and $\mathrm{Re} = \mathrm{Re}_0\varepsilon^{-2}$ for the uniform flow of perfect gas we obtain equation

$$G(\psi) = l\psi_{\xi^\circ\xi^\circ\xi^\circ}, \quad l = M^2(\mathrm{Re}_0\rho_0)^{-1}U\,[1 + (\varkappa - 1)\Pr_0{}^{-1}] \tag{4.1}$$

where $G(\psi)$ represents the left-hand side of Eq. (2.2). Substituting in it

$$\psi = -\frac{2l}{U(\varkappa + 1)} M^{-2} \ln H$$

this equation reduces for $\omega = 0$ to a linear one. The solution

$$u^\circ = r(\lambda) - \frac{2l}{b} s(\lambda)\,\mathrm{th}\,[s(\lambda)\,\xi^\circ - pyr(\lambda)\,s(\lambda) + s_0(\lambda)] \tag{4.2}$$

$$\lambda = 2U\eta_x \quad b = (\varkappa + 1)UM^2$$

for (4.1) of the form derived in [15] defines the structure of a plane shock wave. In this equation r, s, and s_0 are arbitrary functions. Selecting for $s \gtrless 0$ $r = \mp 2l\, b^{-1}s$ we find that $u^\circ \to 0$ when $\xi^\circ \to -\infty$ and $u^\circ \to 2r(\lambda) < 0$ when $\xi^\circ \to \infty$. Joining (2.1) and (4.2) with (1.7) we obtain $\delta = \varepsilon$ and $E_0(\eta) = 2r(\lambda)$.

For $\xi^\circ \to \infty$ asymptotics of (4.1) is of the form (1.2), if $m < 2$ or $m = 2$ and $n \leqslant 0$ (for $m < 2$ or $m = 2$ and $n < 0$ function E is determined, as in the case of (2.2), by (1.3)). This show the possibility of joining expansion (2.1) with

(1.6) in the case of a viscous gas.

The solution of (2.2), (4.1)

$$\psi = \sum_{k=0}^{2} \psi_k (y, t) (\xi^\circ)^k$$

can be used for determining perfect gas flows with a shock wave or a weak discontinuity.

All above results are valid for the trailing edge shock wave which is followed by a uniform flow. In this case it is necessary to substitute in (1.2) $(L - \xi)$ (L is the chord length of the profile whose end point is at $y = 0, x = L$) for ξ, and in formulas in Sects. 2–4 take ξ° to mean the variable $\xi^\circ = (L - \xi)\varepsilon^{-1}$. All formulas, including those for viscous gas, remain unaltered.

5. Let us derive the nonlinear equation of a general form for defining the flow in the neighborhood of weak shock waves which in the linear theory coincide with characteristics. We introduce in the exact equation for $\Phi (x, y, t)$ new variables ξ, η, and θ which depend on x, y, and t, and seek a solution of the obtained equation for flows that differ only slightly from a uniform stream of the form (R is an arbitrary function)

$$\Phi = Ux + \varepsilon^2 \psi (\xi^\circ, \eta, \theta) R (x, y, t) + \ldots, \quad \xi = \varepsilon\xi^\circ, \quad \varepsilon \ll 1 \qquad (5.1)$$

Let us assume that after passing to variables ξ, η, and θ all coefficients in the equation for Φ expressed in terms of variables ξ°, η, and θ are of order unity (this obtains, for example, for variables $\xi = x_1 - f_1 (x_2, x_3)$, $\eta = f_2 (x_2, x_3)$, and $\theta = f_3 (x_2, x_3)$). Retaining leading terms we obtain

$$2K^\circ_{\xi\eta}\psi_{\xi^\circ\eta} + 2K^\circ_{\xi\theta}\psi_{\xi^\circ\theta} + \psi_{\xi^\circ}(L_\xi^\circ + 2R_0^{-1}K^\circ_{\xi R}) - (\varkappa + 1) R_0 M_\xi^\circ \psi_{\xi^\circ}\psi_{\xi^\circ\xi^\circ} = 0 \qquad (5.2)$$

where the small circle in the subscript denotes leading terms of expansions of functions M_ξ, N_ξ, $K_{\xi\eta}$, $K_{\xi\theta}$, $K_{\xi R}$, L_ξ, and R in ε, where

$$M_\xi = (\xi_x^2 + \xi_y^2)(\xi_t + U\xi_x), \quad L_\xi = a^2 \left(\xi_{xx} + \xi_{yy} + \frac{\omega}{y}\xi_y\right) - \xi_{tt} - 2U\xi_{xt} - U^2\xi_{xx}$$

$$K_{\xi\eta} = a^2 (\xi_x\eta_x + \xi_y\eta_y) - (\xi_t + U\xi_x)(\eta_t + U\eta_x)$$

The nontriviality condition

$$N_\xi = a^2 (\xi_x^2 + \xi_y^2) - (\xi_t + U\xi_x)^2 = 0 \qquad (5.3)$$

must then be satisfied. In these formulas $K_{\xi\theta}$ and $K_{\xi R}$ are obtained by the substitution in the expression for $K_{\xi\eta}$ of functions θ and R, respectively, for function η; L_ξ is obtained by substituting in the equation for function φ_1 ξ for function φ_1. The subscripts at M, K, L, and N are markers, while in the remaining cases the letter subscript indicates a derivative. Function R can be used for simplifying (5.2). Equation (5.3) for function $\xi (x, y, t)$ implies that $\xi = \text{const}$ is a characteristic line (surface) of the linear equation for φ_1 $(L_\varphi = 0)$. Functions η and θ are arbitrary and can be selected so as to facilitate the solution of the problem (provided the condition for the order of coefficients is satisfied).

Condition (5.3) may be widened. To obtain a nontrivial equation for ψ it is sufficient to specify that function $\xi(x, y, t)$ determined by the equation $N_\xi = F(\xi, x, y, t)$, in which function F is such that

$$F[\varepsilon\xi^\circ, x(\varepsilon\xi^\circ, \eta, \theta), y(\varepsilon\xi^\circ, \eta, \theta), t(\varepsilon\xi^\circ, \eta, \theta)] = \varepsilon F_0(\xi^\circ, \eta, \theta)$$

i.e. that $N_\xi \sim \varepsilon$ but all remaining coefficients in the equation for Φ are of order unity. Then the equation for ψ assumes the form $F_0\psi_{\xi^\circ\xi^\circ} + G(\psi) = 0$ in which $G(\psi)$ is the left-hand side of Eq. (5.2).

The approximate conditions for (5.2) at the shock wave $\xi^\circ = \xi^\circ(\eta, \theta)$ are of the form

$$\overset{\circ}{K}_{\xi\eta}\frac{\partial\xi^\circ}{\partial\eta} + \overset{\circ}{K}_{\xi\theta}\frac{\partial\xi^\circ}{\partial\theta} + \frac{\varkappa+1}{4}M_\xi^\circ(\psi_{\xi^\circ} + \psi^*_{\xi^\circ}) = 0, \quad \psi = \psi^* \tag{5.4}$$

If $R = 1$, $\xi = x - \beta y$, $\eta = y$, and $\theta = t$, then from (5.2) and (5.4) we obtain (2.2) and (2.3). Setting

$$R = 1, \quad \xi = x \pm (a^2t^2 - y^2)^{1/2}, \quad \eta = y, \quad \theta = t, \quad U = 0, \quad a^2 = (\varkappa+1)/2$$

we obtain the equation which defines the flow in the neighborhood of the sound pulse (or of a weak shock wave) $\xi \approx 0$ propagating through the quiescent or slightly perturbed gas. It can be shown that in problems of propagation of a sound pulse or a weak shock wave a linear expansion of the form (1.6) for $\xi = x \pm (a^2t^2 - y^2)^{1/2}$ also has regions of irregularity. For instance, for $n = 0$ it is irregular when $\xi \sim \delta^{1/(2-m)}$, $m < 2$. Representing the general solution of (5.2) in the form $\xi^\circ = \xi^\circ(u^\circ, y, t)$ it is possible to join the linear and nonlinear expansions. For plane self-similar flows of the form $\Phi = t\Phi^*(\xi/t, y/t)$ we have $m = {}^3/_2$ $(n = 0)$. Equation (5.2) for $\xi = x \pm (a^2t^2 - y^2)^{1/2}$, $\eta = y$, and $\theta = t$ is an analog of the equation for a perfect gas [13] that is obtained from (5.2) for

$$\xi = at - \rho_*, \quad \eta = x = \rho_*\cos\theta_*, \quad \theta = y = \rho_*\sin\theta_*$$
$$R = 1, \quad U = 0$$

it is also an analog of the equation in [10], differing from these only by the form of variables.

We introduce now the expansion

$$\Phi = Ux + \varepsilon^2\psi(\xi^\circ, \eta^\circ, \theta)R(x, y, t) + \ldots, \qquad \xi = \varepsilon\xi^\circ$$
$$\eta = \sqrt{\varepsilon}\eta^\circ, \quad \varepsilon \ll 1$$

Retaining the previous conditions for the coefficients in the equation for Φ, we obtain Eq. (5.2) and condition (5.4) in which it is necessary to substitute

$$N_{\eta^\circ}\psi_{\eta^\circ\eta^\circ}, \quad -\frac{1}{2}\left(\frac{\partial\xi^\circ}{\partial\eta^\circ}\right)^2 N_\eta^\circ$$

for their respective first terms.

In this case besides the condition $N_\xi = 0$ the nontriviality condition $K_{\xi\eta} = 0$ must be satisfied. These conditions may be widened by specifying that functions ξ, η, and θ are to be such that

$$N_\xi = \varepsilon N_\xi^\circ(\xi^\circ, \eta^\circ, \theta) + \ldots, \quad K_{\xi\eta} = \sqrt{\varepsilon}\overset{\circ}{K}_{\xi\eta}(\xi^\circ, \eta^\circ, \theta^\circ) + \ldots$$

The equation for ψ then assumes the form

$$N_{\xi^\circ}\psi_{\xi^\circ\xi^\circ} + N_{\eta^\circ}\psi_{\eta^\circ\eta^\circ} + G(\psi) = 0 \qquad (5.5)$$

where G is the left-hand side of Eq. (5.2). Equation (5.5) is also useful in the calculation of nonlinear gas flows. Thus for $R = U = 1$, $\xi = x$, $\eta = y$, and $\theta = t$ ($N_\xi = K_{\xi\eta} = 0$) we obtain the transonic equation for unstable flows [16, 17], and for $U = 0$, $R = 1$, $\xi = (x - a_0 t) / a_0 t$, $\eta = y / a_0 t$, and $\psi = t\psi_*(\xi, \eta, \ln t)$ we have the equation for short waves [11, 12].

It is interesting that Eq. (5.5) includes the linear equation for φ_I when $N_\xi = N_\theta = K_{\xi\eta} = K_{\eta\theta} = L_\eta = L_\theta = 0$ and $R = 1$. The condition $N_\theta = 0$ means that $\theta = \text{const}$, as well as that $\xi = \text{const}$ are characteristics of the equation for φ_I. Examples of such variables are

$$\xi_1 = (-U \pm a)t + x, \quad \eta_1 = (-U \pm a)t + x + y, \quad \theta_1 = y \pm at \quad (\omega = 0); \quad \xi_2 = (-U \pm a)t + x, \quad \eta_2 = y, \quad \theta_2 = x + (-U \mp at)$$

For these variables the propagation of perturbations from a point source in an unperturbed gas takes place over a circle [17, 18].

Equation (5.5) may be used for calculating the essentially two-dimensional flows (the term $\psi_{\eta^\circ\eta^\circ}$ is present) in the neighborhood of the interaction point of two weak shock waves (e.g., $\xi_1 = \varepsilon\xi^\circ(\eta^\circ, \theta)$ and $\theta_I = \theta(\xi^\circ, \eta^\circ)$) or in the neighborhood of the interaction point of a shock wave (e.g., $\xi_2 = \varepsilon\xi^\circ(\eta^\circ, \theta)$) with a wall (e.g., $\eta_2 = y = \sqrt{\varepsilon}\eta^\circ(\xi^\circ, \theta)$).

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